# Parity-odd and CPT-even electrodynamics of the SME at Finite Temperature

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This work examines the finite temperature properties of the CPT-even and parity-odd electrodynamics of the standard model extension. We start from the partition function written into the functional integral formalism in Ref. [16]. After specializing the Lorentz-violating tensor $W_{\alpha\nu\rho\varphi}$ for the nonbirefringent and parity-odd coefficients, the partition function is explicitly carry out, showing that it is a power of the Maxwell's partition function. Also, it is observed that the LIV coefficients induce an anisotropy in the black body angular energy density distribution. The Planck's radiation law retains its usual frequency dependence and the Stefan-Boltzmann law keeps the same form, except for a global proportionality constant.

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## I. INTRODUCTION

Nowadays, the Standard Model Extension (SME) [1, 2] is the theoretical framework most used to investigate Lorentz invariance violation (LIV). The gauge sector of the SME is composed of a CPT-odd and a CPT-even sector. The CPT-odd one is constituted by the well-known Carroll-Field-Jackiw term [3], whose properties have been well examined in literature [4, 5, 6, 7]. The CPT-even part is represent by a tensor $W^{\alpha\nu\rho\varphi}$ which presents the same symmetries of the Riemann tensor $[W_{\alpha\nu\rho\varphi} = -W_{\nu\alpha\rho\varphi}, W_{\alpha\nu\rho\varphi} = -W_{\alpha\nu\varphi\rho}, W_{\alpha\nu\rho\varphi} = W_{\rho\varphi\alpha\nu}]$ and a double null trace, possessing only 19 independent components.

Recently, the CPT-even has received much attention, yielding the investigation of new electromagnetic phenomena induced by Lorentz violation and the imposition of tight upper bounds on the magnitude of the LIV coefficients. The examination of CPT-even electrodynamics of the SME has started with Kostelecky and Mewes [8] in connection with the study of polarization deviations for light traveling over large cosmological distances [8, 9]. Here, one should also mention other researches involving electromagnetostatics and classical solutions [10, 11, 12, 13], radiation spectrum of the electromagnetic field and CMB [14, 15, 16], photon interactions and quantum electrodynamics processes [17, 18, 19, 20, 21, 22], and synchrotron radiation [23]. A detailed review on the gauge sector of the SME is found in Ref. [24].

At a very recent work [16], we have analyzed the finite temperature behavior of the parity-even part of the CPT-even sector of the SME as an attempt to determine the thermodynamics properties of this electrodynamics. The focus was on the LIV modifications implied on the density energy angular distribution, the Planck radiation law and implications. The partition function was written into the functional integral formalism of Matsubara and explicitly carried out. It was then shown that the altered partition function is a power of the usual Maxwell's partition function. We have then observed that, despite small local fluctuations induced by LIV, the Planck law maintains its usual frequency dependence while the Stefan-Boltzmann retains its usual $T^4$ behavior.

The aim of this present work is to complete the finite temperature analysis for the CPT-even sector, addressing now the contributions of the parity-odd components of the tensor $W_{\alpha\nu\rho\varphi}$ on the thermodynamics of the Maxwell field, searching the modified Planck's law distribution, angular energy density distribution, and Stefan-Boltzmann's laws. We thus follow the same procedure of Ref. [16], taking as starting point the general partition function attained there. Before being explicitly evaluated, this partition function shall be specialized for the case of the nonbirefringent parity-odd coefficients. After explicit evaluation, we show that the modified partition function is a power of the Maxwell usual one, in the very same way as observed for the parity-even case [16].

## II. THE THEORETICAL MODEL AND RESULTS

The CPT-even gauge Lagrangian of the SME is

$$\mathcal{L}=-\frac{1}{4}F_{\alpha\nu}F^{\alpha\nu}-\frac{1}{4}W^{\alpha\nu\rho\varphi}F_{\alpha\nu}F_{\rho\varphi}, \tag{1}$$

where $W^{\alpha\nu\rho\varphi}$ is a renormalizable, dimensionless coupling, composed of 19 elements. In Ref.[16], the Hamiltonian and constraint structure of this electrodynamics was developed using the Dirac method. This analysis allowed to write the correct partition function (in the Matsubara formalism), which integrated on the canonical conjugate momenta and fields has lead to

$$Z\left(\beta\right)=\det\left(-\Box\right)\ \ \left[\det\left(-\Box\delta_{ab}+S_{ab}\right)\right]^{-1/2}. \tag{2}$$

Here, we define the Euclidean operator, $\Box=\partial_a\partial_a=\left(\partial_\tau\right)^2+\nabla^2$, and the symmetric Lorentz-violating operator

$$S_{ab}=2W_{acdb}\partial_c\partial_d.$$

Now, we should particularize the tensor $W_{acdb}$ for the parity-odd sector, which possesses only three nonbirefringent components. This result can be achieved using the parametrization of the tensor $W_{\mu\nu\alpha\beta}$ in terms of four $3\times3$ matrices, $\kappa_{DE},\kappa_{HB},\ \kappa_{DB},\kappa_{HE}$, presented in Refs. [8, 9]:

$$\left(\kappa_{DE}\right)^{jk}=-2W^{0j0k},\left(\kappa_{HB}\right)^{jk}=\frac{1}{2}\epsilon^{jpq}\epsilon^{klm}W^{pqlm},\left(\kappa_{DB}\right)^{jk}=-\left(\kappa_{HE}\right)^{kj}=\epsilon^{kpq}W^{0jpq}. \tag{3}$$

The matrices $\kappa_{DE}$ and $\kappa_{HB}$ represent the parity-even sector and possess together 11 independent components, while $\kappa_{DB}$ and $\kappa_{HE}$ stand for the parity-odd described by 8 components. These four matrices have together the 19 independent elements of the tensor $W_{acdb}$. To isolated the parity-odd sector, we take $\kappa_{DE}=\kappa_{HB}=0$. The parity-odd sector is written in terms of an antisymmetric $(\kappa_{o+})$ and a symmetric matrix $(\widetilde{\kappa}_{o-})$ , given as

$$\left(\widetilde{\kappa}_{o+}\right)_{kj}=\frac{1}{2}(\kappa_{DB}+\kappa_{HE})_{kj},\quad\left(\widetilde{\kappa}_{o-}\right)_{kj}=\frac{1}{2}(\kappa_{DB}-\kappa_{HE})_{kj}. \tag{4}$$

Taking into account the birefringence constraint $\widetilde{\kappa}_{o-}=\frac{1}{2}(\kappa_{DB}-\kappa_{HE})\leq10^{-32}$[8, 9, 25], we obtain $\kappa_{DB}=\kappa_{HE}$. This together the condition $\kappa_{DB}=-\left(\kappa_{HE}\right)^T$ implies that the matrix $\kappa_{DB}=\widetilde{\kappa}_{o+}$ is anti-symmetric (possessing only three components). Such restriction yields only 3 parity-odd nonbirefringent parameters, parameterized in terms of a three-vector $\boldsymbol{\kappa}$ [25]

$$\kappa_j=\frac{1}{2}\epsilon_{jmn}\left(\widetilde{\kappa}_{o+}\right)_{mn}. \tag{5}$$

Into the finite temperature formalism, the matrices (3) are redefined as

$$\left(\kappa_{DE}\right)_{kj}=2W_{\tau k\tau j},\quad\left(\kappa_{HB}\right)_{kj}=\frac{1}{2}\epsilon_{kpq}\epsilon_{jmn}W_{pqmn},\quad\left(\kappa_{DB}\right)_{kj}=-\left(\kappa_{HE}\right)_{jk}=W_{\tau kpq}\epsilon_{jpq}. \tag{6}$$

We should now carry out the determinant of the operator $(-\Box\delta_{ab}+S_{ab})$ in (2) for the three nonbirefringent parity-odd components of the tensor $W_{acdb}$, now written in terms of the $\boldsymbol{\kappa}$ vector as

$$W_{\tau imn}=\frac{1}{2}\left[\kappa_m\delta_{in}-\kappa_n\delta_{im}\right]. \tag{7}$$

For computing such functional determinant, we write this operator (in Fourier space) as $p^2\delta_{ab}-\tilde{S}_{ab}$, where $\tilde{S}_{ab}=2W_{acdb}p_cp_d$. Under the prescription (7), the matrix elements of $\tilde{S}_{ab}$ are

$$\tilde{S}_{\tau\tau}=0,\quad\tilde{S}_{\tau j}=\left(\boldsymbol{\kappa}\cdot\mathbf{p}\right)p_j-\mathbf{p}^2\kappa_j,\quad\tilde{S}_{ij}=-2\left(\boldsymbol{\kappa}\cdot\mathbf{p}\right)p_\tau\delta_{ij}+p_\tau\left(\kappa_ip_j+\kappa_jp_i\right). \tag{8}$$

Thus, the functional determinant is

$$\det\left(-\Box\delta_{ab}+S_{ab}\right)=\det\left(-\Box\right)^2\det\left[-\Box-2\left(\boldsymbol{\kappa}\cdot\nabla\right)\partial_\tau+\boldsymbol{\kappa}^2\nabla^2-\left(\boldsymbol{\kappa}\cdot\nabla\right)^2\right]\det\left[-\Box-2\left(\boldsymbol{\kappa}\cdot\nabla\right)\partial_\tau\right]. \tag{9}$$

Replacing it in the partition function (2), it follows:

$$Z(\beta) = Z_{\kappa}^{(1)}(\beta)\, Z_{\kappa}^{(2)}(\beta)\,, \tag{10}$$

where the quantities, $Z_{\kappa}^{(1)}(\beta)$ and $Z_{\kappa}^{(2)}(\beta)$, are given as

$$Z_{\kappa}^{(1)}(\beta) \;=\; \det\left[-\Box - 2(\boldsymbol{\kappa}\cdot\nabla)\,\partial_\tau + \boldsymbol{\kappa}^2\nabla^2 - (\boldsymbol{\kappa}\cdot\nabla)^2\right]^{-1/2}, \tag{11}$$

$$Z_{\kappa}^{(2)}(\beta) \;=\; \det\left[-\Box - 2(\boldsymbol{\kappa}\cdot\nabla)\,\partial_\tau\right]^{-1/2}, \tag{12}$$

They represent the contributions of the two polarization modes of the modified photon field. Let us observe that if we only consider the first order contribution of the LIV background, $\boldsymbol{\kappa}$, both modes would give the same contribution to the partition function. At leading order, the associated dispersion relations provide nonbirefringence, a result in according with the statements of Refs. [8, 9, 24] and other works that follow this prescription [12, 25]. The explicit evaluation of the dispersion relations is developed in the Appendix.

The computation of the functional determinants is performed using the well-known formulae $\det\hat{\mathrm{O}} = \exp(\mathrm{Tr}\ln\hat{\mathrm{O}})$, thus, we obtain

$$\ln Z_{\kappa}^{(1)}(\beta) \;=\; -\frac{1}{2}\mathrm{Tr}\ln\left[-\Box - 2(\boldsymbol{\kappa}\cdot\nabla)\,\partial_\tau + \boldsymbol{\kappa}^2\nabla^2 - (\boldsymbol{\kappa}\cdot\nabla)^2\right], \tag{13}$$

$$\ln Z_{\kappa}^{(2)}(\beta) \;=\; -\frac{1}{2}\mathrm{Tr}\ln\left[-\Box - 2(\boldsymbol{\kappa}\cdot\nabla)\,\partial_\tau\right]. \tag{14}$$

We can now evaluate the involved trace of expressions (13,14) writing the gauge field in terms of a Fourier expansion,

$$A_a(\tau,\mathbf{x}) = \left(\frac{\beta}{V}\right)^{\frac{1}{2}} \sum_{n,\mathbf{p}} e^{i(\omega_n\tau + \mathbf{x}\cdot\mathbf{p})}\tilde{A}_a(n,\mathbf{p}), \tag{15}$$

where $V$ designates the system volume and $\omega_n$ are the bosonic Matsubara's frequencies, $\omega_n = \dfrac{2n\pi}{\beta}$, for $n = 0, 1, 2, \cdots$.

In this way, the contributions of the two modes of the gauge field are expressed as

$$\ln Z_{\kappa}^{(1)}(\beta) \;=\; -\frac{1}{2}V\int\frac{d^3\mathbf{p}}{(2\pi)^3}\sum_{m=-\infty}^{+\infty}\ln\beta^2\left[(\omega_m)^2 + \mathbf{p}^2 + 2(\boldsymbol{\kappa}\cdot\mathbf{p})\,\omega_m - \boldsymbol{\kappa}^2\mathbf{p}^2 + (\boldsymbol{\kappa}\cdot\mathbf{p})^2\right], \tag{16}$$

$$\ln Z_{\kappa}^{(2)}(\beta) \;=\; -\frac{1}{2}V\int\frac{d^3\mathbf{p}}{(2\pi)^3}\sum_{m=-\infty}^{+\infty}\ln\beta^2\left[(\omega_m)^2 + \mathbf{p}^2 + 2(\boldsymbol{\kappa}\cdot\mathbf{p})\,\omega_m\right]. \tag{17}$$

For evaluating the integrals, we first implement the translation $\mathbf{p} \to \mathbf{p} - \omega_m\boldsymbol{\kappa}$. We then use spherical coordinates, $\mathbf{p} = \omega(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)\,, \boldsymbol{\kappa}\cdot\mathbf{p} = \kappa\omega\cos\theta,\ \omega = |\mathbf{p}|\,,\ \kappa = |\boldsymbol{\kappa}|$. By performing the summation in $n$, and doing the respective rescalings in the variable $\omega$, we obtain the following expressions:

$$\ln Z_{\kappa}^{(1)} \;=\; -\frac{V}{(2\pi)^3}\left(1-\kappa^2\right)^{3/2}\int d\Omega\frac{1}{\left(1-\kappa^2\sin^2\theta\right)^{3/2}}\int_0^\infty d\omega\;\omega^2\ln\left(1-e^{-\beta\omega}\right), \tag{18}$$

$$\ln Z_{\kappa}^{(2)} \;=\; -\frac{V}{(2\pi)^3}\left(1-\kappa^2\right)^{3/2}\int d\Omega\;\int_0^\infty d\omega\;\omega^2\ln\left(1-e^{-\beta\omega}\right), \tag{19}$$

where $d\Omega = \sin\theta d\theta d\phi$ is the solid-angle element.

Then, the partition function for the parity-odd sector of the CPT-even electrodynamics of the SME is

$$\ln Z = -\frac{V}{(2\pi)^3}\left(1-\kappa^2\right)^{3/2}\int d\Omega\left[1 + \frac{1}{\left(1-\kappa^2\sin^2\theta\right)^{3/2}}\right]\int_0^\infty d\omega\;\omega^2\ln\left(1-e^{-\beta\omega}\right). \tag{20}$$

The dependence on $\theta$ shows that the LIV interaction yields an anisotropic character for the angular distribution of the energy density. By performing the $\omega-$integration in (20), we achieve the energy density per solid-angle element,

$$u(\beta,\Omega) \;=\; \frac{\pi}{120\beta^4}\left(1-\kappa^2\right)^{3/2}\left[1 + \frac{1}{\left(1-\kappa^2\sin^2\theta\right)^{3/2}}\right], \tag{21}$$

which reveals the anisotropy induced by the LIV coefficient (the power angular spectrum is maximal in the plane perpendicular to background direction). At leading order, the anisotropy factor is quadratic in the $\boldsymbol{\kappa}-$vector,

$$u(\beta,\Omega) \approx \frac{\pi}{120\beta^4}\left[2+\kappa^2\left(\frac{3}{2}\sin^2\theta-3\right)\right]. \tag{22}$$

This result should be contrasted with the linear contribution induced by anisotropic contribution stemming from the parity-even sector [16].

By performing the angular integrations in Eq. (20), we find that the partition function can be written as

$$Z=(Z_A)^{\gamma(\kappa)}, \tag{23}$$

where $Z_A$ is the partition function of the Maxwell's electrodynamics,

$$\ln Z_A=-\frac{V}{\pi^2}\int_0^\infty d\omega\ \omega^2\ln\left(1-e^{-\beta\omega}\right)=V\frac{\pi^2}{45\beta^3}. \tag{24}$$

and the exponent $\gamma(\kappa)$ is a pure function of the LIV parameter

$$\gamma(\kappa)=\left(1-\kappa^2\right)^{1/2}\left(1-\frac{1}{2}\kappa^2\right). \tag{25}$$

The result (23) for the nonbirefringent and parity-odd components of the tensor $W_{\alpha\beta\mu\nu}$ is similar to one obtained in Ref. [16] for the nonbirefringent and parity-even components.

Starting from the equations (20) or (23), it is easy to derive the modified Planck's radiation law or the modifications in the Stefan-Boltzmann's law, respectively, given as follows:

$$u(\omega)=\gamma(\kappa)\frac{1}{\pi^2}\frac{\omega^3}{e^{\beta\omega}-1}, \quad u=\gamma(\kappa)\frac{\pi^2}{15}T^4. \tag{26}$$

Explicitly, we can observe that the LIV modifications consists in a global multiplicative function which contain all the LIV correction, this way, the Planck's radiation law maintains its functional dependence in the frequency (in all orders in $\kappa$). Similarly, the energy density or the Stefan-Boltzmann law retains its usual temperature dependence ($u\propto T^4$) whereas the Stefan-Boltzmann constant is globally altered as $\sigma\rightarrow\gamma(\kappa)\sigma$, with $\gamma(\kappa)$ given by Eq. (25).

## III. CONCLUSIONS AND REMARKS

In this work we have concluded the study of the finite temperature behavior of the CPT-even and LIV electrodynamics of the SME which was started in Ref. [16]. We have specialized our analysis for the nonbirefringent components of the parity-odd sector of the tensor $W_{\alpha\nu\rho\varphi}$. We have exactly computed the partition function, (23), showing that it is a power of the partition function of the Maxwell electrodynamics as well, being the power a pure function the LIV parameters. Consequently, the Planck's radiation law retains its known functional dependence in the frequency whereas the Stefan-Boltzmann's law keeps the $T^4$-behavior, apart from a multiplicative global factor. It was observed that the LIV interaction induces an anisotropic angular distribution for the black body energy density. A similar behavior was obtained for the nonbirefringent anisotropic components of the parity-even sector [16]. These results show that the partition function of the full CPT-even sector is expressed as a power of the Maxwell's one. This pattern, however, is not shared by the CPT-odd partition function evaluated in Ref.[14]. This difference is ascribed to the dimensional character of the LIV coefficient $k_{AF}$.

## APPENDIX A: DISPERSION RELATIONS

In this Appendix, we write the dispersion relations for this CPT-even and parity-odd electrodynamics as a procedure to confirm the evaluation of the associated partition function. It is important to point out that the dispersion relations

of the parity-odd case may be read off directly from the arguments of the partition functions (13,14) making use of the prescription $\Box \to -p^2, \nabla \to -i\mathbf{p}$, $\partial_\tau \to -ip_0$, which yields

$$\left[p^2 + 2p_0\left(\boldsymbol{\kappa}\cdot\mathbf{p}\right)\right] = 0, \tag{A1}$$

$$\left[p^2 + 2p_0\left(\boldsymbol{\kappa}\cdot\mathbf{p}\right) - \boldsymbol{\kappa}^2\mathbf{p}^2 + \left(\boldsymbol{\kappa}\cdot\mathbf{p}\right)^2\right] = 0. \tag{A2}$$

These dispersion relations can be also obtained straightforwardly from the Maxwell equations for this sector (see Ref. [12]):

$$\begin{aligned} \nabla\cdot\mathbf{E} &= -\kappa\cdot\left(\nabla\times\mathbf{B}\right), && \text{(A3)}\\ \nabla\times\mathbf{B} - \partial_t\left(\mathbf{B}\times\kappa\right) &= \partial_t\mathbf{E} - \nabla\times\left(\mathbf{E}\times\kappa\right), && \text{(A4)}\\ \nabla\cdot\mathbf{B} &= 0\ , && \text{(A5)}\\ \nabla\times\mathbf{E} &= -\partial_t\mathbf{B}. && \text{(A6)} \end{aligned}$$

Writing the electric and magnetic fields in a Fourier representation, $\mathbf{B}(\mathbf{r}) = (2\pi)^{-3}\int\widetilde{\mathbf{B}}(\mathbf{p})\exp(-i\mathbf{p}\cdot\mathbf{r})d^3\mathbf{p}$, $\mathbf{E}(\mathbf{r}) = (2\pi)^{-3}\int\widetilde{\mathbf{E}}(\mathbf{p})\exp(-i\mathbf{p}\cdot\mathbf{r})d^3\mathbf{p}$, the Maxwell equations take on the following form (at the absence of sources):

$$\mathbf{p}\cdot\widetilde{\mathbf{E}} = -\boldsymbol{\kappa}\cdot\left(\mathbf{p}\times\widetilde{\mathbf{B}}\right), \tag{A7}$$

$$\mathbf{p}\times\widetilde{\mathbf{B}} + p_0\left(\widetilde{\mathbf{B}}\times\boldsymbol{\kappa}\right) + p_0\widetilde{\mathbf{E}} = -\mathbf{p}\times\left(\widetilde{\mathbf{E}}\times\boldsymbol{\kappa}\right), \tag{A8}$$

$$\mathbf{p}\times\widetilde{\mathbf{E}} - p_0\widetilde{\mathbf{B}} = 0,\ \ \mathbf{p}\cdot\widetilde{\mathbf{B}} = 0. \tag{A9}$$

From these expressions, it is attained an equation for the electric field components, $M^{jl}\widetilde{E}^l = 0$, where

$$M^{jl} = [p^lp^j - p_0p^j\kappa^l - p_0p^l\kappa^j + \delta^{lj}(p^2 + 2p_0A)]. \tag{A10}$$

where $A = \boldsymbol{\kappa}\cdot\mathbf{p}$. Such operator can be represented as a $3\times 3$ matrix,

$$M^{jl} = \begin{bmatrix} p^2 + 2p_0A + p_1^2 - 2p_0p_1\kappa_1 & p_1p_2 - p_0p_1\kappa_2 - p_0p_2\kappa_1 & p_1p_3 - p_0p_1\kappa_3 - p_0p_3\kappa_1 \\ p_1p_2 - p_0p_1\kappa_2 - p_0p_2\kappa_1 & p^2 + 2p_0A + p_2^2 - 2p_0p_2\kappa_2 & p_2p_3 - p_0p_2\kappa_3 - p_0p_3\kappa_2 \\ p_1p_3 - p_0p_1\kappa_3 - p_0p_3\kappa_1 & p_2p_3 - p_0p_2\kappa_3 - p_0p_3\kappa_2 & p^2 + 2p_0A + p_3^2 + 2p_0p_3\kappa_3 \end{bmatrix}. \tag{A11}$$

After suitable simplification, the determinant of this matrix takes the form

$$\det M^{jl} = p_0^2\left(p^2 + 2Ap_0\right)\left(p^2 + 2Ap_0 - \mathbf{p}^2\boldsymbol{\kappa}^2 + A^2\right). \tag{A12}$$

The condition $\det M^{jl} = 0$ provides the non-trivial solutions for Eq. (A9) and the associated dispersion relations of this model, attained without any approximation. This alternative procedure confirms the correctness of dispersion relations (A1, A2) and of the expressions (13, 14), written at the finite temperature formalism. The intricate character of the relations (A1, A2), involving both $p_0$ and $\mathbf{p}$, imply the following dispersion relations:

$$\begin{aligned} \omega_\pm &= -\left(\boldsymbol{\kappa}\cdot\mathbf{p}\right) \pm \sqrt{\mathbf{p}^2 + \left(\boldsymbol{\kappa}\cdot\mathbf{p}\right)^2}, && \text{(A13)}\\ \omega_\pm &= -\left(\boldsymbol{\kappa}\cdot\mathbf{p}\right) \pm \sqrt{\mathbf{p}^2(1+\boldsymbol{\kappa}^2)}, && \text{(A14)} \end{aligned}$$

which are different even at leading order in $\kappa$. Assuming $|\kappa| <<1$, the expressions (A13,A14) are reduced to the form:

$$\omega_+ = |\mathbf{p}| - \left(\boldsymbol{\kappa}\cdot\mathbf{p}\right), \tag{A15}$$

$$\omega_- = |\mathbf{p}| + \left(\boldsymbol{\kappa}\cdot\mathbf{p}\right). \tag{A16}$$

Here, the root $\omega_+ = |\mathbf{p}| - (\boldsymbol{\kappa}\cdot\mathbf{p})$ represents a positive frequency mode, since $|\kappa| <<1$. On the other hand, the mode $\omega_- = (|\mathbf{p}| + (\boldsymbol{\kappa}\cdot\mathbf{p}))$ stands for the positive energy of an anti-particle (after reinterpretation). This is a negative

frequency mode. It should be mentioned that, despite the double sign in the dispersion relations (A15,A16), they yield the same phase velocities for waves traveling at the same direction. Note the the positive and negative frequency modes are associated with waves which propagate in opposite directions and the term $(\boldsymbol{\kappa}\cdot\mathbf{p})$ changes of the signal under the direction inversion $(\mathbf{p}\rightarrow-\mathbf{p})$. This result confirms the nonbirefringent character of the coefficient $\boldsymbol{\kappa}$ at leading order, as properly stated in Refs. [8, 9, 24], and others [12, 25]. These same dispersion relations can obtained by means of a general evaluation for the dispersion relations (see Appendix of Ref. [16]) or by means of the poles of the gauge propagator of this electrodynamics (see Ref. [26]).

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[1] V. A. Kostelecky and S. Samuel, Phys. Rev. Lett. **63**, 224 (1989); **66**, 1811 (1991); Phys. Rev. D **39**, 683 (1989); **40**, 1886 (1989); V. A. Kostelecky and R. Potting, Nucl. Phys. B **359**, 545 (1991); Phys. Lett. B **381**, 89 (1996); V. A. Kostelecky and R. Potting, Phys. Rev. D **51**, 3923 (1995).

[2] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997); D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).

[3] S. M. Carroll, G. B. Field and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).

[4] A.A. Andrianov and R. Soldati, Phys. Rev. D **51**, 5961 (1995); Phys. Lett. B **435**, 449 (1998); A.A. Andrianov, R. Soldati and L. Sorbo, Phys. Rev. D **59**, 025002 (1998); M. Frank and I. Turan, Phys. Rev. D **74**, 033016 (2006); A. Hariton and R. Lehnert, Phys. Lett. A **367**, 11 (2007).

[5] C. Adam and F. R. Klinkhamer, Nucl. Phys. B **607**, 247 (2001); Phys. Lett. B **513**, 245 (2001); H. Belich, M. M. Ferreira Jr, J.A. Helayel-Neto, M. T. D. Orlando, Phys. Rev. D **67**, 125011 (2003); -ibid, Phys. Rev. **D 69**, 109903 (E) (2004).

[6] R. Jackiw and V. A. Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999); J. M. Chung and B. K. Chung *Phys. Rev.* D **63**, 105015 (2001); J.M. Chung, Phys. Rev. D **60**, 127901 (1999); G. Bonneau, Nucl.Phys. B **593,** 398 (2001); M. Perez-Victoria, Phys. Rev. Lett. **83**, 2518 (1999); M. Perez-Victoria, J. High. Energy Phys. **04**, (2001) 032; O.A. Battistel and G. Dallabona, Nucl. Phys. B **610**, 316 (2001); O.A. Battistel and G. Dallabona, J. Phys. G **28**, L23 (2002); J. Phys. G **27**, L53 (2001); A. P. B. Scarpelli, M. Sampaio, M. C. Nemes, and B. Hiller, Phys. Rev. D **64**, 046013 (2001); T. Mariz, J.R. Nascimento, E. Passos, R.F. Ribeiro and F.A. Brito, J. High. Energy Phys. **0510** (2005) 019; J. R. Nascimento, E. Passos, A. Yu. Petrov, F. A. Brito, J. High. Energy Phys. **0706**, (2007) 016; B. Altschul, Phys. Rev. D **70**, 101701 (2004); A.P.B. Scarpelli, M. Sampaio, M.C. Nemes, B. Hiller, Eur. Phys. J. C **56**, 571 (2008); J. Alfaro, A. A. Andrianov, M. Cambiaso, P. Giacconi, R. Soldati, Phys. Lett. B **639**, 586 (2006); J. Alfaro, A. A. Andrianov, M. Cambiaso, P. Giacconi, R. Soldati, *Induced Lorentz and CPT invariance violations in QED*, arXiv: 0904.3557.

[7] R. Lehnert and R. Potting, Phys. Rev. Lett. **93**, 110402 (2004); Phys. Rev. D **70**, 125010 (2004); Phys. Rev. D **70**, 129906(E) (2004); B. Altschul, Phys. Rev. D **75**, 105003 (2007); Phys. Rev. Lett. **98**, 041603 (2007); B. Altschul, Nucl. Phys. B **796**, 262 (2008); C. Kaufhold and F.R. Klinkhamer, Nucl. Phys. B **734, 1** (2006); C. Kaufhold and F.R. Klinkhamer, Phys. Rev. D **76**, 025024 (2007).

[8] V. A. Kostelecky and M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001); V. A. Kostelecky and M. Mewes, Phys. Rev. D **66**, 056005 (2002).

[9] V. A. Kostelecky and M. Mewes, Phys. Rev. Lett. **97**, 140401 (2006); Astrophys. J. Lett. **689**, L1 (2008).

[10] Q. G. Bailey and V. A. Kostelecky, Phys. Rev. D **70**, 076006 (2004).

[11] X. Xue and J. Wu, Eur. Phys. J. C **48**, 257 (2006); H. Belich, M. M. Ferreira Jr, J.A. Helayel-Neto, M. T. D. Orlando, Phys. Rev. **D 68**, 025005 (2003); H. Belich, T. Costa-Soares, M.M. Ferreira Jr., J. A. Helayël-Neto, Eur. Phys. J. C **42**, 127 (2005); M. B. Cantcheff, Eur. Phys. J. C **46**, 247 (2006); S.O. Vellozo, J.A. Helayel-Neto, A. W. Smith, and L.P.G. de Assis, Int. J. Theor. Phys. **47**, 2934 (2008); Rodolfo Casana, Manoel M. Ferreira Jr and Carlos E. H. Santos, Phys. Rev. D **78**, 025030 (2008).

[12] Rodolfo Casana, Manoel M. Ferreira Jr and Carlos E. H. Santos, Phys. Rev. D **78**, 105014 (2008).

[13] R. Casana, M. M. Ferreira Jr, A. R. Gomes and P. R. D. Pinheiro, Eur. Phys. J. C **62**, 573 (2009).

[14] R. Casana, M. M. Ferreira Jr and J. S. Rodrigues, Phys. Rev. D **78**, 125013 (2008).

[15] J.M. Fonseca, A.H. Gomes, W.A.Moura-Melo, Phys. Lett. B **671**, 280 (2009).

[16] R. Casana, M. M. Ferreira Jr, J. S. Rodrigues and M. R. O. Silva, *Finite Temperature behavior of the CPT-even and parity-even electrodynamics of the Standard Model Extension*, arXiv:0907.1924, to appear in Phys. Rev. D (2009).

[17] V. A. Kostelecky and A. G. M. Pickering, Phys. Rev. Lett. **91**, 031801 (2003); B. Altschul, Phys.Rev. D **70,** 056005 (2004).

[18] C.D. Carone, M. Sher, and M. Vanderhaeghen, Phys. Rev. D **74**, 077901 (2006); B. Altschul, Phys. Rev. D **79**, 016004 (2009).

[19] M.A. Hohensee, R. Lehnert, D. F. Phillips, R. L. Walsworth, Phys. Rev. D **80**, 036010(2009); M.A. Hohensee, R. Lehnert, D. F. Phillips, R. L. Walsworth, Phys. Rev. Lett. **102**, 170402 (2009).

[20] C. Adam and F. R. Klinkhamer, Nucl. Phys. B **657**, 214 (2003).

[21] F.R. Klinkhamer and M. Risse, Phys. Rev. D **77**, 016002 (2008); F.R. Klinkhamer and M. Risse, Phys. Rev. D **77**, 117901 (A) (2008).

[22] F.R. Klinkhamer and M. Schreck, Phys. Rev. D **78**, 085026 (2008).

[23] R. Montemayor and L.F. Urrutia, Phys. Rev. D **72**, 045018 (2005); B. Altschul, Phys. Rev. D **72**, 085003 (2005); B. Altschul, Phys. Rev. D **74**, 083003 (2006); B. Altschul, Phys. Rev. D **72**, 085003 (2005).

[24] V. A. Kostelecky and M. Mewes, Phys. Rev. D **80**, 015020 (2009).

[25] A. Kobakhidze and B.H.J. McKellar, Phys. Rev. D **76**, 093004 (2007).

[26] R. Casana, M. M. Ferreira Jr, A. R. Gomes, P. R. D. Pinheiro, *Gauge propagator and physical consistency of the CPT-even part of the Standard Model Extension*, arXiv:0909.0544.

[27] J. I. Kapusta, *Finite-Temperature Field Theory*, Cambridge University Press, Cambridge, 1989 ; M. Le Bellac, *Thermal Field Theory*, Cambridge University Press, Cambridge, 1996.